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FREE CONVECTION HEAT TRANSFER TO STEAM UNDER VARIABLE PROPERTY CONDITIONS

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WHEN convective heat transfer takes place under conditions where there are large temperature differences within the fluid, it is then necessary to consider the effects of variable fluid properties. One approach to predicting the heat transfer in such a situation is to employ an equation that corresponds to a constant-property flow and to evaluate the fluid properties appearing therein at a suitable reference temperature. Rules for choosing the reference temperature are generally deduced by comparing the results from a limited number of solutions of the variable property problem with the predictions of the constant-property solution. Reference temperature rules for a range of flow situations have been published in the literature.

This paper is concerned specifically with the free convection flow of water vapor adjacent to an isothermal vertical plate. The variable-property free-convection problem for ideal gases having relatively simple temperature-dependent viscosities and thermal conductivities has been previously investigated by Sparrow and Gregg [1]. However, water vapor does not, in general, obey the ideal gas law, nor are its transport properties governed by relations similar to those for other gases. Therefore, there is reason to suspect that the Sparrow–Gregg reference-temperature rule may not be applicable to water vapor. The present investigation is aimed at determining a reference-temperature rule appropriate to water vapor.

In carrying out the present analysis, the temperature dependencies of all the participating thermodynamic and transport properties were included. Furthermore, the properties also change with pressure level and this, too, was fully taken into account.

The governing equations of the free-convection problem, embodying the boundary layer assumptions, may be stated as follows:

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \pm g(\rho_\infty - \rho) + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \tag{2}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right). \tag{3}$$

The symbols are standard and need not be defined here. The subscript ∞ denotes conditions in the ambient fluid, while the subscript w denotes conditions at the wall. When $T_w > T_\infty$, the + sign is affixed to the buoyancy force in equation (2) and the free-convection flow is upward. When $T_w < T_\infty$, the − sign is affixed and the flow is downward.

By employing the following similarity transformation

$$\eta = \frac{c}{x^{\frac{1}{4}}}\int_0^y \frac{\mathrm{d}y}{\varphi_\mu}, \qquad \psi = 4c\nu_\infty x^{\frac{3}{4}} f(\eta), \qquad \frac{T}{T_\infty} = \theta(\eta) \tag{4}$$

with $c = (g/4\nu_\infty^2)^{\frac{1}{4}}$, the conservation equations become

$$\left(\frac{f'}{\varphi_\mu\varphi_\rho}\right)'' + 3f\left(\frac{f'}{\varphi_\mu\varphi_\rho}\right)' - \frac{2f'^2}{\varphi_\mu\varphi_\rho} \pm \varphi_\mu(1 - \varphi_\rho) = 0 \tag{5}$$

$$\left(\frac{\varphi_k}{\varphi_\mu}\theta'\right)' + 3Pr_\infty\varphi_c\theta' = 0 \tag{6}$$

in which φ represents a property ratio such that $\varphi_i = i/i_\infty$ and $Pr = c_p\mu/k$. The primes denote differentiation with respect to η. The foregoing equations are to be solved subject to the boundary conditions that

$$f(0) = f'(0) = 0, \qquad \theta(0) = \theta_w; \qquad f'(\infty) = 0, \qquad \theta(\infty) = 1. \tag{7}$$

The fluid properties k, μ, and c_p were taken from the NBS tabulation [2], while the density ρ was evaluated from equation (13) of the Steam Tables [3].

Numerical solutions of the governing differential equations were carried out for a wide range of temperature boundary conditions (prescribed T_w and T_∞) and pressure levels. These solutions supplied a pool of heat-transfer results which, in conjunction with the constant-property

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results, were employed in constructing a reference-temperature rule. For the variable-property solutions, the pressure level was varied from 0·04 to 25 atm in increments indicated by the curve parameters of Figs. 1 and 2. At each of these pressure levels, as many as 18 different pairs of thermal boundary conditions were investigated. The extreme temperature differences were $T_w = 1685$ degR and $T_\infty = T_{sat} + 20$ degR on the one hand, and $T_w = T_{sat} + 20$ degR and $T_\infty = 1685$ degR on the other, where T_{sat} is the saturation temperature corresponding to the pressure level.

The constant-property heat-transfer relationship for which we are seeking a reference-temperature rule is

$$Nu = \left(\frac{GrPr}{4}\right)^{\frac{1}{4}} \Phi(Pr) \tag{8}$$

in which

$$Nu = \frac{hx}{k}, \quad h = \frac{q}{|T_w - T_\infty|}, \quad Gr = \frac{g\beta|T_w - T_\infty|x^3}{\nu^2}, \quad Pr = \frac{c_p\mu}{k}. \tag{9}$$

The quantity Φ is a slowly-varying function of Pr which, in the range appropriate to steam, is expressable as

$$\Phi(Pr) = 0{\cdot}4748 + 0{\cdot}1251Pr - 0{\cdot}0328Pr^2. \tag{10}$$

As written, equation (8) pertains to the local rate of heat flux per unit area q; however, by affixing a multiplying factor of $\frac{4}{3}$ to the right-hand side, the same equation applies for the average heat-transfer rate Q.

The constant-property solution, equation (8), was numerically evaluated for the same thermal boundary conditions as were specified for the numerical solutions of the variable-property problem. The fluid properties appearing in equation (8) were calculated at various temperatures between the prescribed values of T_w and T_∞. The authors were unable to find a detailed tabulation of the expansion coefficient β in the literature. It was therefore necessary to prepare such a tabulation as part of this investigation. This was accomplished by a 5-point numerical differentiation of density values determined from equation (13) of the Steam Tables. The information thus obtained is shown in Fig. 1. The ordinate variable was chosen in cognizance of the fact that $\beta^{\frac{1}{4}}$ is required in evaluating equation (8). Inasmuch as $\beta T = 1$ for an ideal gas, it is seen that there are large deviations from the ideal gas behavior near the saturation line.

Some systematic procedure for evaluating the properties was sought which would lead to a close correspondence between the heat-transfer results predicted by the constant-property and variable-property solutions. On the basis of careful comparisons, it was found that the following rule was most suitable

$$T^* = T_w + 0{\cdot}46(T_\infty - T_w) \tag{11a}$$

for c_p, μ, k, and ρ

$$\beta = \beta(T_\infty) \tag{11b}$$

in which T^* is the reference temperature. It should be emphasized that while equation (11b) indicates that β is to be evaluated at T_∞, it is not to be inferred that $\beta = 1/T_\infty$, rather, β is to be taken from Fig. 1.

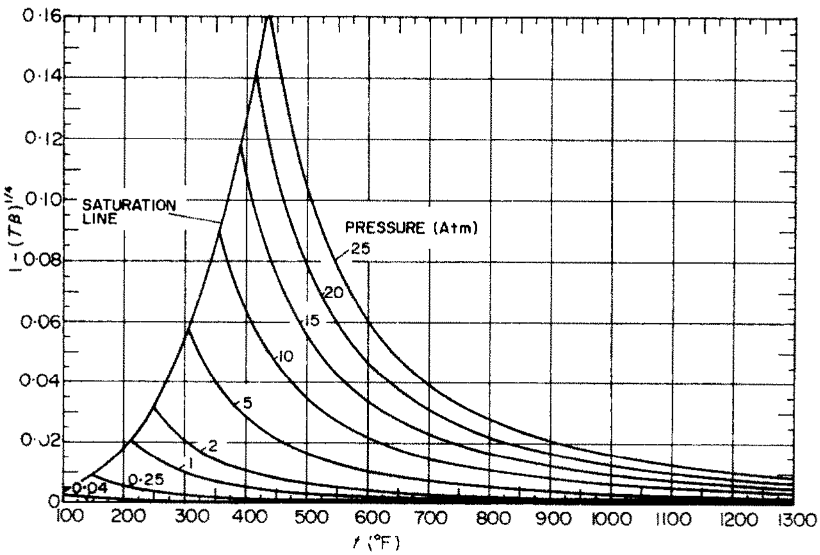

FIG. 1. Thermal expansion coefficient for steam (β in °R^{-1}, T in °R).

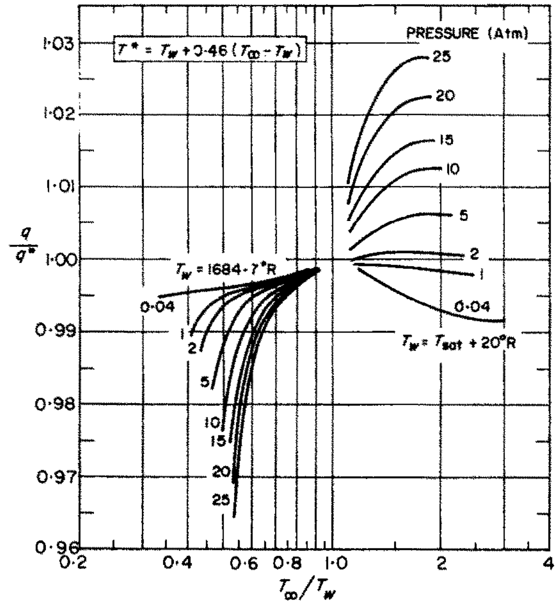

FIG. 2. Comparison of constant-property and variable-property heat-transfer predictions.

It is instructive to display the effectiveness with which the reference temperature brings the constant-property heat-transfer predictions into correspondence with those from the variable-property solutions. Figure 2 has been prepared in this connection. The ordinate is the ratio of the heat fluxes q and q^* which correspond respectively to the variable-property and the constant-property solutions. The thermal boundary conditions and the pressure level are indicated on the abscissa and on the grid of the figure. Thermal conditions other than those shown on the figure were also studied, but are omitted because they do not provide as demanding a test of the reference-temperature rule.

From an inspection of the figure, it is evident that for low and moderate pressures, the constant-property heat flux values q^* are within one per cent of those from the variable-property solutions. The greatest deviations of q^* from q that are shown in the figure are about three per cent, but these are for rather extreme temperature conditions at relatively high pressures.

To force even greater fidelity of the constant-property predictions, a multiplicative correction factor has been fitted as follows

$$\left[1 - 0{\cdot}001814p\left(\frac{T_\infty}{T_w} - 1\right)\right]. \qquad (12)$$

This bracketed quantity is to be appended as a multiplier of the right-hand side of equation (8). When the reference temperature specification of equations (11a) and (11b) is used in conjunction with this correction factor, the constant-property relation, equation (8) provides heat-transfer results that are accurate to within one per cent over the entire range investigated.

As a final matter, it is of interest to inquire as to how effective is the reference temperature rule of Sparrow and Gregg when applied to water vapor. The Sparrow–Gregg rule specifies a value of 0·38 in lieu of 0·46 in equation (11a) and replaces equation (11b) with the specification that $\beta = 1/T_\infty$. By careful scrutiny of the computed results, it was ascertained that the change from 0·46 to 0·38 gives rise to changes in q^* that are less than one per cent. However, the evaluation of β as $1/T_\infty$ can, under some conditions, lead to significant errors. Since β enters equation (8) as $\beta^{\frac{1}{4}}$, the extent of these errors is directly discernable from Fig. 1. It is seen from this figure that at the higher pressures, errors of the order of ten per cent are possible when T_∞ is in the neighborhood of the saturation state.

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